Enrollment No: _____ Exam Seat No: _____

C.U.SHAH UNIVERSITY

Summer Examination-2016

Subject Name: Engineering Mathematics-II
Subject Code: 4TE02EMT2 Branch: B.Tech(All)
Semester: 2 Date: 09/05/2016 Time: 10:30 to 1:30 Marks: 70

Instructions:

(1) Use of Programmable calculator & any other electronic instrument is prohibited.
(2) Instructions written on main answer book are strictly to be obeyed.
(3) Draw neat diagrams and figures (if necessary) at right places.
(4) Assume suitable data if needed.

Q-1 Attempt the following questions: (14)

a) $\int_{-\pi/2}^{\pi/2} \sin^7 x \; dx =$ _____

(a) 0 (b) 1 (c) $\frac{\pi}{2}$ (d) $\frac{1}{2}$

b) $\int_{0}^{\pi/2} \sqrt{1-\frac{1}{4}\sin^2\theta}\; d\theta =$ _____

(a) $E\left(\frac{1}{2}\right)$ (b) $E\left(\frac{1}{4}\right)$ (c) $K\left(\frac{1}{2}\right)$ (d) $K\left(\frac{1}{4}\right)$

c) $\int_{0}^{1}\int_{0}^{\sqrt{x}} dy\; dx =$ _____

(a) $\frac{1}{2}$ (b) $\frac{2}{3}$ (c) 0 (d) y

d) The value of $\int_{-\pi}^{\pi} \sin mx \sin nx \; dx$ for $m \neq \pm n$ is

(a) 2π (b) π (c) $\frac{\pi}{2}$ (d) 0

e) $\beta\left(\frac{1}{2},\frac{1}{2}\right) =$ _____.

(a) $\sqrt{\pi}$ (b) 1 (c) 0 (d) π

f) $\Gamma(n)\Gamma(1-n) =$ _____

(a) $\dfrac{\pi}{\cos n\pi}$ (b) $\dfrac{\sqrt{\pi}\,\Gamma(2n)}{2^{2n-1}}$ (c) $\dfrac{\sqrt{\pi}\,\Gamma(n)}{2^{2n}}$ (d) $\dfrac{\pi}{\sin n\pi}$

g) The curve $y^2(2a-x) = x^3$ represents

(a) Cissoid of Diocle (b) Witch of Agnesi
(c) Folium of Descartes (d) Strophoid

h) The curve passes through the origin, if the equation does not contain _____

(a) terms in x (b) terms in y (c) constant term (d) none of these

i) Length of curve for $x = f(y)$ is defined by

(a) $\int_{x_1}^{x_2} \sqrt{\left[1+\left(\dfrac{dy}{dx}\right)\right]}\,dx$ (b) $\int_{x_1}^{x_2} \sqrt{\left[1+\left(\dfrac{dy}{dx}\right)^2\right]}\,dx$

(c) $\int_{y_1}^{y_2} \sqrt{\left[1+\left(\dfrac{dx}{dy}\right)^2\right]}\,dx$ (d) $\int_{y_1}^{y_2} \sqrt{\left[1+\left(\dfrac{dx}{dy}\right)\right]}\,dx$

j) $\int_0^2\int_2^4\int_4^6 dx\,dy\,dz =$ _____

(a) 1 (b) 6 (c) 4 (d) 8

k) The order of the differential equation $\dfrac{d^2y}{dx^2} = \left[1+\left(\dfrac{dy}{dx}\right)^3\right]^{\frac{2}{3}}$ is

(a) 1 (b) 2 (c) 3 (d) 6

l) The equation $P(x,y)\,dx - Q(x,y)\,dy = 0$ is exact if

(a) $P_x = Q_y$ (b) $P_y = Q_x$ (c) $P_x = -Q_y$ (d) $P_y = -Q_x$

m) The series $1-\dfrac{1}{2}+\dfrac{1}{4}-\dfrac{1}{8}+\ldots$ is convergent then sum of the series

(a) 1 (b) 2 (c) $\dfrac{1}{2}$ (d) none of these

n) The series $1-\dfrac{1}{2^2}+\dfrac{1}{3^2}-\dfrac{1}{4^2}+\ldots$ is

(a) convergent (b) divergent (c) oscillatory (d) none of these

Attempt any four questions from Q-2 to Q-8

Q-2 Attempt all questions

a) Find the volume common to the cylinder $x^2 + y^2 = a^2$ and $x^2 + z^2 = a^2$. **(05)**

b) Evaluate: $\int_0^{\pi} x \sin^8 x \cos^6 x \, dx$ **(05)**

c) Solve: $\frac{dy}{dx} - x^3 = \frac{3y}{x}, \; y(1) = 4$ **(04)**

Q-3 Attempt all questions

a) Evaluate: $\int_{-\infty}^{\infty} e^{-h^2x^2} \, dx$ **(05)**

b) Prove that (i) $erf_c(-x) = 2 - erf_c(x)$ **(05)**

(ii) $erf(-x) = -erf(x)$

c) Test for the convergence the series $\sum_{n=1}^{\infty} \frac{\lfloor (n+1)x \rfloor^n}{n^{n+1}}$ **(04)**

Q-4 Attempt all questions

a) Find the radius of convergence and interval of the series $\sum_{n=1}^{\infty} \frac{x^n}{\sqrt{n}}$. **(05)**

b) Trace the curve $r^2 = a^2 \cos 2\theta$. **(05)**

c) Prove that $\int_0^1 \left(\frac{x}{1-x^3} \right)^{\frac{1}{2}} dx = \frac{\pi}{3}$. **(04)**

Q-5 Attempt all questions

a) Evaluate: $\int_0^{\log 2} \int_0^x \int_0^{x+y} e^{x+y+z} \, dz \, dy \, dx$ **(05)**

b) Solve: $\frac{dy}{dx} + x \sin 2y = x^2 \cos^2 y$ **(05)**

c) Test for convergence the series $4 - 1 + \frac{1}{4} - \frac{1}{16} + \ldots$ and if it is convergent then also find its sum. **(04)**

Q-6 Attempt all questions

a) Find the area bounded by the parabola $y^2 = 4x$ and the line $2x - 3y + 4 = 0$. **(05)**

b) Prove that $\int_0^a x^5 \left(2a^2 - x^2\right)^{-3} dx = \frac{1}{2}\left(\log 2 - \frac{1}{2}\right)$. **(05)**

c) Find the orthogonal trajectories of the family of parabola $ay^2 = x^3$. **(04)**

Q-7 Attempt all questions

a) Change the order of integration and evaluate $\int_0^a \int_{\frac{x}{a}}^{\sqrt{\frac{x}{a}}} \left(x^2 + y^2\right) dx\, dy$. **(05)**

b) In a circuit containing resistance R, inductance L, the voltage E and the current i are connected in series. Given that $L = 640$ henries, $R = 250$ ohms, $E = 500$ volts and $i = 0$ when $t = 0$. Find the time that elapses before i reach 90% of its maximum value. **(05)**

c) Solve: $2xy\,dy - \left(x^2 + y^2 + 1\right) dx = 0$ **(04)**

Q-8 Attempt all questions

a) Evaluate: $\int_0^\infty \frac{dx}{\sqrt{x^2 + 16}\sqrt{x^2 + 25}}$ **(05)**

b) Trace the curve $y^2\left(a + x\right) = x^2\left(a - x\right)$. **(05)**

c) Find the whole length of the lemniscates of Bernoulli $r^2 = a^2 \cos 2\theta$. **(04)**